

Calculators may be used in this examination
provided they are not capable of being used
to store alphabetical information other than
hexadecimal numbers

UNIVERSITY OF BIRMINGHAM

School of Computer Science

LC Data Structures and Algorithms

Main Summer Examinations 2023

Time allowed: 2 hours

[Answer all questions]

Note

Answer ALL questions. Each question will be marked out of 20. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Question 1

- (a) If a Binary Tree is either empty or is a node with a left child and a right child that are both Binary Trees, what additional conditions must a Binary Tree satisfy for it to be

- (i) *Complete*, and
- (ii) a *Binary Heap Tree*?

[4 marks]

- (b) The items [6, 9, 4, 8, 12, 5] need to be inserted one at a time into a *Binary Heap Tree*, starting from an empty tree. Show the state of the tree after each item has been inserted.

[6 marks]

- (c) One approach for sorting an array of items, a , is to first insert them into a *Binary Search Tree*, t , and then output them back into the array in order. Write, in pseudocode, a recursive procedure for filling the array from the *Binary Search Tree*, and specify what the initial call of the procedure is. You may call any of the standard primitive operators for binary trees

[10 marks]

Question 2

- (a) Describe how one can establish the best possible average-case time complexity of any comparison-based sorting algorithm. Does *Binary Search Tree*-based sorting achieve that? **[6 marks]**
- (b) Show how *Two-Phase Radix Sort* can be used to sort the following set of dates given in day/month format: [17/7, 12/7, 8/4, 8/7, 9/3, 12/4, 17/3, 12/6]. **[6 marks]**
- (c) Explain how *Radix Sort* could be used to sort a set of integers without duplicates. Describe the time complexity of your approach and determine if it is more efficient than the best possible comparison-based sorting algorithms. **[8 marks]**

Question 3

- (a) Consider a graph represented by a symmetric $N \times N$ weight matrix. What does the **size** and **symmetry** of that matrix indicate?

In the weight matrix, the symbol ∞ is used to represent the lack of an edge connecting the vertices indicated by the row and column where the symbol appears, and the **connectivity level** of a graph is defined as the actual number of edges divided by the maximum possible number of edges.

Given a symmetric $N \times N$ weight matrix representing a graph, provide a formula for the connectivity level of the graph as a function of N and the number M of ∞ symbols it contains? **[6 marks]**

- (b) Describe an efficient greedy edge-based algorithm for determining a minimal spanning tree of a weighted graph. In what sense is your algorithm greedy? **[6 marks]**

- (c) What aspect of the algorithm you describe in part (b) above contributes most to its time complexity, and what is this algorithm's overall time complexity? Comment on the speed of this algorithm on highly connected graphs compared to Jarník-Prim's vertex-based algorithm for the same problem. **[8 marks]**

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches must be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.